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Simple and Accurate Solutions of the Scattering Coefficients of E-Plane Junctions in Rectangular Waveguides

Anton Widarta, Shuzo Kuwano, and Kinchi Kokubun

***Abstract*—Simple and accurate solution of the scattering coefficients of the E-plane right-angle bend in rectangular waveguide is presented. The solution is obtained by the mode-matching method in which the electromagnetic fields in waveguides are matched with those in junction section formed by a sectoral region. In the same procedure, the solutions of the scattering coefficients of the E-plane T-junction and the cross junction can be also obtained easily. By using the numerical results, the scattering properties of the dominant modes and higher-order modes in the E-plane right-angle bend are examined in detail.**

I. Introduction

Rectangular waveguide junctions such as the right-angle bend, the T-junction and the cross junction are representatives of fundamental microwave circuits, which are applied to such as filters, multiplexers [1], and power dividers [2]. It is desirable that the scattering properties of these junctions are analyzed rigorously and that the obtained solutions are simple, convenient, and highly accurate over a wide frequency range.

The modeling of the above waveguide junctions is a canonical problem, and numbers of analysis methods have been proposed. Marcuvitz [3] represented the waveguide junction by an equivalent circuit. However, since the solution is an approximate one, there are limits in usable bandwidth and accuracy. In order to improve the equivalent circuits in [3], Lampariello and Oliner [4] presented new equivalent circuits for open and slit-coupled T-junctions. In [5], the T-junction boundary value problem is solved using equivalent circuit concept and leading to a calculation of the equivalent admittance matrix. The full numerical analyses, such as the finite-element method [6] and the boundary-element method [7], are useful techniques but require advanced computer processing.

On the other hand, the mode-matching method is a very efficient method for the analysis of these problems. However, since the electromagnetic fields in each junction section cannot be expanded in terms of the modal functions of a rectangular waveguide, some procedures are necessary in applying this method. Lewin [8] proposed a new expansion function in which the junction section of the right-angle bend is considered to be a region separated from the waveguides, though he did not derive the solution. In [9], the junction of the right-angle bend is divided into certain regions, and their boundaries are appropriately shorted. Then, the electromagnetic fields in the region are expanded in terms of the modal functions of a rectangular coordinate system. Similar strategies have been used to analyze the T-junction in [1], [2], and the hybrid junction in [10]. In the BCMM [11], instead of the point-matching [12], the contour-integral matching method in [13] is applied for the rigorous analysis of cascade arbitrarily shaped H-plane discontinuities in rectangular waveguides.

This paper presents a simple and accurate solution of the scattering coefficients of the E-plane right-angle bend in rectangular waveguide. The solution is obtained by the mode-matching method in which the electromagnetic fields in waveguide regions are matched with those in junction section formed by a sectoral region [14], [15] at the circumference boundary. This solution is expressed succinctly in form of matrix, and the formulations of the matrix elements are directly given. Hence, the solution is very simple and convenient. Since the solution is obtained by rigorous analysis, the obtained numerical results are highly accurate over a wide frequency range. In the same procedure, the solutions of the scattering coefficients of the E-plane T-junction and the cross junction can also be obtained easily. By using the numerical results, the scattering properties of the dominant modes and higher-order modes in the E-plane waveguide right-angle bend are examined in detail.

II. Theory

The rectangular waveguide I (region I) of width a and height b_1 and the rectangular waveguide II (region II) of width a and height b_2 are orthogonally joined to form an E-plane right-angle bend as shown in Fig. 1(a). Consider a TE_{10} mode is incident from region I on the junction, and LSE_{1n} $(n = 0, 1, 2, 3, \ldots)$ modes are scattered back into regions I and II. The field expansions are given as follows:

The incident wave is

$$H_x^{i\tau} = A^i \sin\left(\frac{\pi}{a}x_\tau\right)e^{j\beta^i z_\tau}. \tag{1}$$

The scattered wave is

$$H_x^{\nu} = \sum_{n=0}^{\infty} A_{1n}^{\nu} \sin\left(\frac{\pi}{a}x_\nu\right)\cos\left(\frac{n\pi}{b_\nu}y_\nu\right)e^{-j\beta_{1n}^{\nu} z_\nu} \tag{2}$$

where the indices τ and ν characterize the incidence region and

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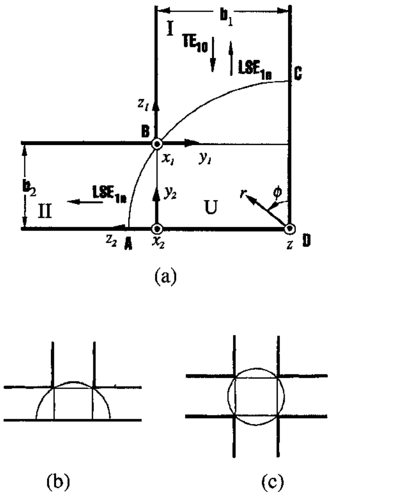


Fig. 1. E-plane junctions in a rectangular waveguide. (a) Right-angle bend. (b) T-junction. (c) Cross junction.

scattering region, respectively, and

$$n = 0, 1, 2, 3, \ldots, \quad \beta_{1n}^{\nu} = \sqrt{(k_c)^2 - \left(\frac{n\pi}{b_\nu}\right)^2}$$
$$k_c = \beta^i = \sqrt{\omega^2 \varepsilon_0 \mu_0 - \left(\frac{\pi}{a}\right)^2} \tag{3}$$

where ω denotes the angular frequency, ε_0 and μ_0 are the permittivity and the permeability of free space. A^i and A_{1n}^{ν} are the amplitude coefficients of the incident and scattered waves, respectively. The other field components are derived from (1) and (2), respectively.

Now, the junction section is divided into a quarter of a circular region (region U), as shown in Fig. 1(a), and the fields are expanded in terms of a cylindrical TE mode function. The H_z^U component is given as follows

$$H_z^U = \sum_{m=0}^{\infty} G_m J_m(k_c r) \cos(m\phi) \sin\left(\frac{\pi}{a} z\right) \tag{4}$$

where $m = 0, 2, 4, 6, \ldots, J_m$ is the Bessel function of the first kind and m-th order, and G_m is the expansion coefficient. The other field components are derived from this expression.

The electromagnetic fields of each region must satisfy the continuity condition at the boundary (arc ABC). By applying the orthogonality relations of electromagnetic fields of region U at the boundaries, the infinite sets of linear equations in the unknown amplitude coefficients A_{1n}^1 and A_{1n}^2 are obtained. Practically, in order to find the solution, the higher-order terms of modal function expansions are truncated according to the required accuracy, yielding a finite number of linear equations. The solution can be expressed as follows

$$\begin{bmatrix} A_{1n}^1 \\ \cdots \\ A_{1n}^2 \end{bmatrix} = \left[P_{pq}^1 \ \vdots \ P_{pq}^2\right]^{-1} \left[P_P^{i1} A^i\right] \tag{5}$$

where

$$\begin{aligned} P_{pq}^{\nu} = &-\frac{j\beta_{1n}^{\nu}}{J_m'(k_c r_0)\beta^i} \int_{S1}^{S2} \cos\left(\frac{n\pi}{b_\nu} y_\nu\right) \\ &\times \cos\phi_\nu \cos(m\phi) e^{-j\beta_{1n}^{\nu} z_\nu} d\phi + \frac{\left(\frac{n\pi}{b_\nu}\right)}{J_m'(k_c r_0)\beta^i} \\ &\times \int_{S1}^{S2} \sin\left(\frac{n\pi}{b_\nu} y_\nu\right) \sin\phi_\nu \cos(m\phi) e^{-j\beta_{1n}^{\nu} z_\nu} d\phi \\ &- \frac{1}{J_m(k_c r_0)} \int_{S1}^{S2} \cos\left(\frac{n\pi}{b_\nu} y_\nu\right) \cos(m\phi) e^{-j\beta_{1n}^{\nu} z_\nu} d\phi \end{aligned} \tag{6}$$

$$\begin{aligned} P_p^{i\tau} = &-\frac{j}{J_m'(k_c r_0)} \int_{S1}^{S2} \cos\phi_\tau \cos(m\phi) e^{j\beta^i z_\tau} d\phi \\ &+ \frac{1}{J_m(k_c r_0)} \int_{S1}^{S2} \cos(m\phi) e^{j\beta^i z_\tau} d\phi \end{aligned} \tag{7}$$

$$\left.\begin{array}{l} A^i = 1, r_0 = \sqrt{b_1^2 + b_2^2}. \\ \text{For } v = 1 \text{ or } \tau = 1, \\ \quad \phi_1 = \phi \\ \quad y_1 = b_1 - r_0 \sin\phi, z_1 = r_0 \cos\phi - b_2 \\ \quad S_1 = 0, S_2 = \sin^{-1} \frac{b_1}{r_0} \\ \text{For } v = 2, \\ \quad \phi_2 = \phi - \frac{\pi}{2} \\ \quad y_2 = r_0 \cos\phi, z_2 = r_0 \sin\phi - b_1 \\ \quad S_1 = \sin^{-1} \frac{b_1}{r_0}, S_2 = \frac{\pi}{2} \\ \quad p = 1, 2, 3, \ldots, 2t, q = 1, 2, 3, \ldots, t \\ \quad m = 2(p-1), n = q - 1 \end{array}\right\} \tag{8}$$

where t is the number of terms in the modal expansions of the scattered waves in the rectangular waveguides.

From (2) and (5), the magnitudes and the phases of the power scattering coefficients of the dominant and higher-order waves of each waveguide can be expressed as follows

$$\left.\begin{array}{ll} \left|S_{1n}^{11}\right|^2 = \frac{\beta_{1n}^1 |A_{1n}^1|^2}{2^\delta \beta^i |A^i|^2}, & \left|S_{1n}^{21}\right|^2 = \frac{b_2 \beta_{1n}^2 |A_{1n}^2|^2}{2^\delta b_1 \beta^i |A^i|^2} \\ 2\theta_{1n}^{11} = 2\arg \frac{A_{1n}^1}{A^i}, & 2\theta_{1n}^{21} = 2\arg \frac{A_{1n}^2}{A^i} \end{array}\right\} \tag{9}$$

where $\delta = 0$ for $n = 0$, and $\delta = 1$ for $n \neq 0$.

When t increases, these coefficients improve in accuracy and the magnitudes satisfy the energy conservation as shown in the next expression

$$\sum_{n=0}^{\infty} \left|S_{1n}^{11}\right|^2 + \sum_{n=0}^{\infty} \left|S_{1n}^{21}\right|^2 = 1. \tag{10}$$

The solutions of the scattering coefficients of the E-plane T-junction [Fig. 1(b)] and the cross junction [Fig. 1(c)] can be obtained by the same procedure in which the electromagnetic fields in waveguides are matched with those in each junction section formed by a semicircular and a circular region, respectively, at each circumference boundary.

III. Numerical Results

All numerical values have been obtained with a FUJITSU M1700/6 (7 MIPS) mainframe computer using double precision arithmetic and the integrations of (6) and (7) have been calculated numerically by means of the Clenshaw-Curtis integration method [16].

Table I shows the convergence of the scattering coefficients. The normalized frequency is $ka/\pi = 1.5$, and the waveguide geometry is $b_1 = b_2 = a/2$. $|S_{10}^{q1}|^2$ is the scattering coefficient of the dominant wave, while G expresses the satisfaction of the energy conservation (10). From this table, it is found that the accuracy improves as t increases. For instance, at $t = 10$, it converges with three significant figures. Then, the cpu time is less than 2 sec. However, in a higher frequency range, the convergence is slowed, and hence, the number of expansion terms needs to be increased according to the required accuracy.

Fig. 2 compares the scattering coefficients obtained by the present method with the results in [9]. From this figure, it is found that the results of the present method agree well with those in [9], both in magnitudes and phases.

Fig. 3 shows the scattering coefficients from standard frequency range to higher frequency range. From Fig. 3(a), it is found that

TABLE I

CONVERGENCE OF THE SCATTERING COEFFICIENTS FOR THE NUMBER OF TERMS IN MODAL FUNCTION EXPANSIONS ($b_1 = b_2 = a/2$, $ka/\pi = 1.5$)

t	$\lvert S^{11}_{10}\rvert^2$	$\lvert S^{21}_{10}\rvert^2$	G
4	0.297689	0.702311	1.000000
6	0.300514	0.699486	1.000000
8	0.301600	0.698400	1.000000
10	0.302139	0.697861	1.000000
12	0.302450	0.697550	1.000000

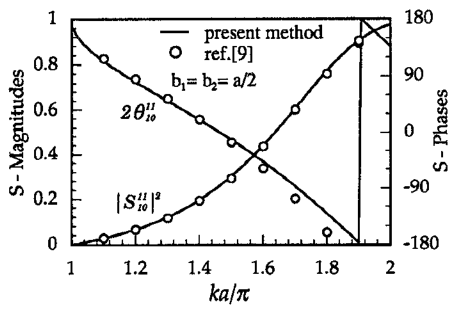
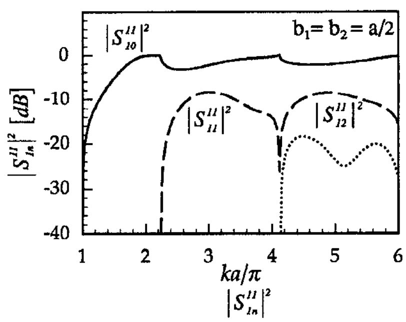
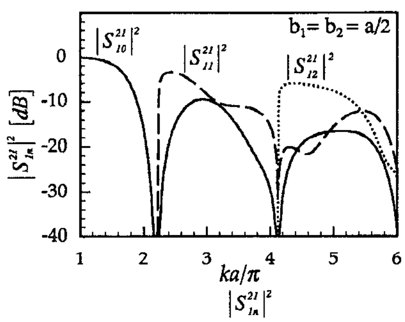


Fig. 2. Comparison of the results in this paper with those in [9].



(a)



(b)

Fig. 3. Scattering properties of E-plane right-angle bend in a rectangular waveguide.

the graph of the reflection coefficient $|S^{11}_{10}|^2$ of the dominant mode constricts at the cutoff frequencies of the higher-order modes and the values increase along ka/π. Moreover, the reflection coefficients of the lower-order modes are larger than the higher-order modes. The transmission coefficients $|S^{21}_{1n}|^2$ exhibit antiresonances over ka/π and decrease along ka/π, as shown in Fig. 3(b).

IV. CONCLUSION

In this paper, the scattering coefficients of the E-plane right-angle bend in rectangular waveguide have been obtained by the mode-matching method. The solution has the following advantages:

1) The solution is expressed succinctly in form of matrix and the formulations of the matrix elements are directly given. Therefore, it is convenient to use and the numerical results are also simply obtained.
2) The accuracy of the numerical results are able to be improved by increasing the number of terms in modal expansions over a wide frequency range.

In the same procedure, the solutions of the scattering coefficients of the E-plane T-junction and the cross junction can be also obtained easily.

By using the numerical results, the scattering properties of the dominant modes and higher-order modes in the E-plane waveguide right-angle bend are examined in detail.

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